

Cosmological Constraints on Variable Warm Dark Matter

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ABSTRACT

Although Λ CDM model is very successful in many aspects, it has been seriously challenged. Recently, warm dark matter (WDM) remarkably rose as an alternative of cold dark matter (CDM). In the literature, many attempts have been made to determine the equation-of-state parameter (EoS) of WDM. However, in most of the previous works, it is usually assumed that the EoS of dark matter (DM) is constant (and usually the EoS of dark energy is also constant). Obviously, this assumption is fairly restrictive. It is more natural to assume a variable EoS for WDM (and dark energy). In the present work, we try to constrain the EoS of variable WDM with the current cosmological observations. We find that the best fits indicate WDM, while CDM is still consistent with the current observational data. However, Λ CDM is still better than WDM models from the viewpoint of goodness-of-fit. So, in order to distinguish WDM and CDM, the further observations on the small/galactic scale are required. On the other hand, in this work we also consider WDM whose EoS is constant, while the role of dark energy is played by various models. We find that the cosmological constraint on the constant EoS of WDM is fairly robust.

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I. INTRODUCTION

Nowadays, dark energy has become one of the most active fields in physics and astronomy, since the great discovery of the current accelerated expansion of our universe in 1998 [1, 20]. On the other hand, dark matter (DM) was invoked to interpret the rotation curves of spiral galaxies for many years [2]. Since it is well known that hot dark matter (HDM) cannot be competent for the cosmological structure formation, cold dark matter (CDM) has become the leading candidate. In fact, the well-known Λ CDM model has been established as the standard model in cosmology today [1].

Although Λ CDM model is very successful in many aspects, recently it has been seriously challenged. According to the brief reviews in e.g. [3], these serious challenges to Λ CDM model include, for example, (1) Λ CDM predicts significantly smaller amplitude and scale of large-scale velocity flows than observations; (2) Λ CDM predicts fainter Type Ia supernova (SNIa) at high redshift z ; (3) Λ CDM predicts more dwarf or irregular galaxies in voids than observed; (4) Λ CDM predicts shallow low concentration and density profiles of cluster haloes in contrast to observations; (5) Λ CDM predicts galaxy halo mass profiles with cuspy cores and low outer density while observations indicate a central core of constant density and a flattish high dark mass density outer profile; (6) Λ CDM predicts a smaller fraction of disk galaxies due to recent mergers expected to disrupt cold rotationally supported disks. Even when one replaces the cosmological constant Λ with other (dynamical) dark energy candidates, these challenges still cannot be successfully addressed. In particular, the main source of the challenges on the small/galactic scale might be CDM. We refer to e.g. [3] for details.

On the other hand, recently warm dark matter (WDM) remarkably rose as an alternative of CDM. The leading WDM candidates are the keV scale sterile neutrinos. In fact, the keV scale WDM is an intermediate case between the eV scale HDM and the GeV scale CDM. Unlike CDM which is challenged on the small/galactic scale (as mentioned above), it is claimed that WDM can successfully reproduce the astronomical observations over all the scales (from small/galactic to large/cosmological scales) [4]. The key is the connection between the mass of DM particles and the free-streaming length ℓ_{fs} (structure smaller than ℓ_{fs} will be erased). The eV scale HDM is too light and hence all structures below Mpc scale will be erased; the GeV scale CDM is too heavy and hence the structures below kpc scale cannot be erased (therefore CDM is challenged on the small/galactic scale); in between, the keV scale WDM works well [4]. We refer to e.g. [4] for a comprehensive review.

It is well known that the equation-of-state parameter (EoS) plays an important role in cosmology. In particular, the EoS of CDM and radiation/HDM are 0 and 1/3, respectively. In between, the EoS of WDM, w_m , satisfies $0 \leq w_m \leq 1/3$ obviously. Of course, the realistic value of w_m should be determined from the astronomical observations. A non-zero w_m indicates WDM, rather than CDM. In the literature, many attempts have been made to determine the EoS of DM. For example, in [5], assuming a constant w_m and the role of dark energy is played by a cosmological constant Λ whose EoS is -1 exactly, it is found that $-1.50 \times 10^{-6} < w_m < 1.13 \times 10^{-6}$ if DM produces no entropy, and $-8.78 \times 10^{-3} < w_m < 1.86 \times 10^{-3}$ if the adiabatic sound speed vanishes. Note that in [5] the allowed range of the EoS of DM was relaxed and could be negative, but this is somewhat unnatural and hence it is not the case which will be investigated in the present work. Following the method proposed in [6] which suggested to measure the EoS of DM by combining kinematic and gravitational lensing data, the authors of [7] found that the EoS of DM is consistent with pressureless dark matter within the errors. The authors of [8] considered the observational constraints on a cosmological model with variable EoS of matter and dark energy, namely, $w_m = 1/[3(x^\alpha + 1)]$ and $w_{de} = \bar{w}x^\alpha/(x^\alpha + 1)$, where $x \equiv a/a_*$ with a_* being a reference value of the scale factor a , and $\alpha > 0$ is a constant model parameter. The model considered in [8] was proposed to unify the radiation dominated phase and the dark energy dominated phase. The matter behaves as radiation when $x \ll 1$ and DM when $x \gg 1$. Note that this motivation is not for WDM, and its EoS w_m , w_{de} are ad hoc in some sense. The authors of [9] considered the cosmological constraints on WDM whose EoS is a constant, while the EoS of dark energy is also a constant. They claimed that the cosmological data favor $w_m = 0.006 \pm 0.001$ (suggesting WDM), and $w_{de} = -1.11 \pm 0.03$ (corresponding to phantom dark energy). The CDM whose EoS $w_m = 0$ and the cosmological constant Λ whose EoS $w_{de} = -1$ are disfavored beyond 3σ confidence level.

Note that in most of the previous works, it is usually assumed that the EoS of DM is constant (and usually the EoS of dark energy is also constant). Obviously, this assumption is fairly restrictive. It is more natural to assume a variable EoS for WDM (and dark energy). In fact, this is the main subject

of the present work. We will try to constrain the EoS of variable WDM with the current cosmological observations. In Sec. II, we briefly introduce the observational data used in this work. In Sec. III, we consider the cosmological constraints on WDM whose EoS is variable. In fact, we adopt the familiar Chevallier-Polarski-Linder (CPL) parameterization for WDM, namely, $w_m = w_{m0} + w_{ma}(1 - a)$. Unlike the somewhat ad hoc parameterization in [8], noting that the Taylor series expansion of any function $F(x)$ is given by $F(x) = F(x_0) + F_1(x - x_0) + (F_2/2!)(x - x_0)^2 + (F_3/3!)(x - x_0)^3 + \dots$, the CPL parameterization for WDM can be regarded as the Taylor series expansion of w_m with respect to the scale factor a up to first order (linear expansion), and hence it is naturally motivated. In this section, we consider three cases, namely, the role of dark energy is played by a cosmological constant Λ , and dark energy described by constant EoS, CPL parameterized EoS, respectively. In Sec. IV, noting that in most of the previous works constant EoS of both WDM and dark energy are assumed, here we also consider WDM whose EoS is constant, but dark energy is a cosmological constant Λ , or dark energy whose EoS are constant and CPL parameterized, respectively. We try to see whether the cosmological constraint on the constant EoS of WDM is robust for various types of dark energy, especially when the EoS of dark energy is variable. In Sec. V, some concluding remarks are given.

II. OBSERVATIONAL DATA

Recently, the Supernova Cosmology Project (SCP) Collaboration released the Union2.1 compilation which consists of 580 Type Ia supernovae (SNIa) [10]. The Union2.1 compilation is the largest published and spectroscopically confirmed SNIa sample to date. The data points of the 580 Union2.1 SNIa compiled in [10] are given in terms of the distance modulus $\mu_{obs}(z_i)$. On the other hand, the theoretical distance modulus is defined as

$$\mu_{th}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0, \quad (1)$$

where $\mu_0 \equiv 42.38 - 5 \log_{10} h$ and h is the Hubble constant H_0 in units of 100 km/s/Mpc, whereas

$$D_L(z) = (1 + z) \int_0^z \frac{d\tilde{z}}{E(\tilde{z}; \mathbf{p})}, \quad (2)$$

in which \mathbf{p} denotes the model parameters; z is the redshift; $E \equiv H/H_0$, in which $H \equiv \dot{a}/a$ is the Hubble parameter; a dot denotes the derivative with respect to cosmic time t ; $a = (1 + z)^{-1}$ is the scale factor (we have set $a_0 = 1$; the subscript “0” indicates the present value of corresponding quantity). Correspondingly, the χ^2 from 580 Union2.1 SNIa is given by

$$\chi_\mu^2(\mathbf{p}) = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i)]^2}{\sigma^2(z_i)}, \quad (3)$$

where σ is the corresponding 1σ error. The parameter μ_0 is a nuisance parameter but it is independent of the data points. One can perform a uniform marginalization over μ_0 . However, there is an alternative way. Following [11, 12], the minimization with respect to μ_0 can be made by expanding the χ_μ^2 of Eq. (3) with respect to μ_0 as

$$\chi_\mu^2(\mathbf{p}) = \tilde{A} - 2\mu_0\tilde{B} + \mu_0^2\tilde{C}, \quad (4)$$

where

$$\begin{aligned} \tilde{A}(\mathbf{p}) &= \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0, \mathbf{p})]^2}{\sigma_{\mu_{obs}}^2(z_i)}, \\ \tilde{B}(\mathbf{p}) &= \sum_i \frac{\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0, \mathbf{p})}{\sigma_{\mu_{obs}}^2(z_i)}, \quad \tilde{C} = \sum_i \frac{1}{\sigma_{\mu_{obs}}^2(z_i)}. \end{aligned}$$

Eq. (4) has a minimum for $\mu_0 = \tilde{B}/\tilde{C}$ at

$$\tilde{\chi}_\mu^2(\mathbf{p}) = \tilde{A}(\mathbf{p}) - \frac{\tilde{B}(\mathbf{p})^2}{\tilde{C}}. \quad (5)$$

Since $\chi_{\mu, \min}^2 = \tilde{\chi}_{\mu, \min}^2$ (up to a constant) obviously, we can instead minimize $\tilde{\chi}_\mu^2$ which is independent of μ_0 . In addition to SNIa, the other useful observations include the cosmic microwave background (CMB) anisotropy [13] and the large-scale structure (LSS) [14]. However, using the full data of CMB and LSS to perform a global fitting consumes a large amount of computation time and power. As an alternative, one can instead use the shift parameter R from CMB, and the distance parameter A of the measurement of the baryon acoustic oscillation (BAO) peak in the distribution of SDSS luminous red galaxies. In the literature, the shift parameter R and the distance parameter A have been used extensively. It is argued in e.g. [15] that they are model-independent and contain the main information of the observations of CMB and BAO, respectively. As is well known, the shift parameter R of CMB is defined by [15, 16]

$$R \equiv \Omega_{m0}^{1/2} \int_0^{z_*} \frac{d\tilde{z}}{E(\tilde{z})}, \quad (6)$$

where the redshift of recombination $z_* = 1091.3$ which was determined by the Wilkinson Microwave Anisotropy Probe 7-year (WMAP7) data [13], and $\Omega_{m0} \equiv 8\pi G\rho_{m0}/(3H_0^2)$ is the present fractional density of matter. The shift parameter R relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at z_* and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations [15, 16]. The value of R has been determined to be 1.725 ± 0.018 from the WMAP7 data [13]. On the other hand, the distance parameter A of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies [14] is given by

$$A \equiv \Omega_{m0}^{1/2} E(z_b)^{-1/3} \left[\frac{1}{z_b} \int_0^{z_b} \frac{d\tilde{z}}{E(\tilde{z})} \right]^{2/3}, \quad (7)$$

where $z_b = 0.35$. In [17], the value of A has been determined to be $0.469 (n_s/0.98)^{-0.35} \pm 0.017$. Here the scalar spectral index n_s is taken to be 0.963, which comes from the WMAP7 data [13]. So, the total χ^2 is given by

$$\chi^2 = \tilde{\chi}_\mu^2 + \chi_{CMB}^2 + \chi_{BAO}^2, \quad (8)$$

where $\tilde{\chi}_\mu^2$ is given in Eq. (5), $\chi_{CMB}^2 = (R - R_{obs})^2/\sigma_R^2$ and $\chi_{BAO}^2 = (A - A_{obs})^2/\sigma_A^2$. The best-fit model parameters are determined by minimizing the total χ^2 . As in [18–20], the 68.3% confidence level is determined by $\Delta\chi^2 \equiv \chi^2 - \chi_{\min}^2 \leq 1.0, 2.3, 3.53, 4.72$ and 5.89 for $n_p = 1, 2, 3, 4$ and 5 respectively, where n_p is the number of free model parameters. Similarly, the 95.4% confidence level is determined by $\Delta\chi^2 \equiv \chi^2 - \chi_{\min}^2 \leq 4.0, 6.18, 8.02, 9.72$ and 11.31 for $n_p = 1, 2, 3, 4$ and 5 , respectively.

III. COSMOLOGICAL CONSTRAINTS ON VARIABLE WDM

In this section, we consider the cosmological constraints on WDM whose EoS is variable. Throughout this work, we consider a flat Friedmann-Robertson-Walker (FRW) universe which contains dark energy and WDM (we assume that radiation and the baryon component can be ignored). Here, we adopt the familiar CPL parameterization for WDM, namely, its EoS is given by

$$w_m = w_{m0} + w_{ma}(1 - a), \quad (9)$$

where w_{m0} and w_{ma} are both constants. Unlike the somewhat ad hoc parameterization used in [8], as mentioned in Sec. I, the CPL parameterization for WDM can be regarded as the Taylor series expansion of w_m with respect to the scale factor a up to first order (linear expansion), and hence it is naturally motivated. To ensure $0 \leq w_m \leq 1/3$ in the whole history ($0 \leq a \leq 1$) as mentioned in Sec. I, we require that

$$0 \leq w_{m0} \leq 1/3, \quad 0 \leq w_{m0} + w_{ma} \leq 1/3. \quad (10)$$

In this section, we consider three cases, namely, the role of dark energy is played by a cosmological constant Λ , and dark energy described by constant EoS, CPL parameterized EoS, respectively.

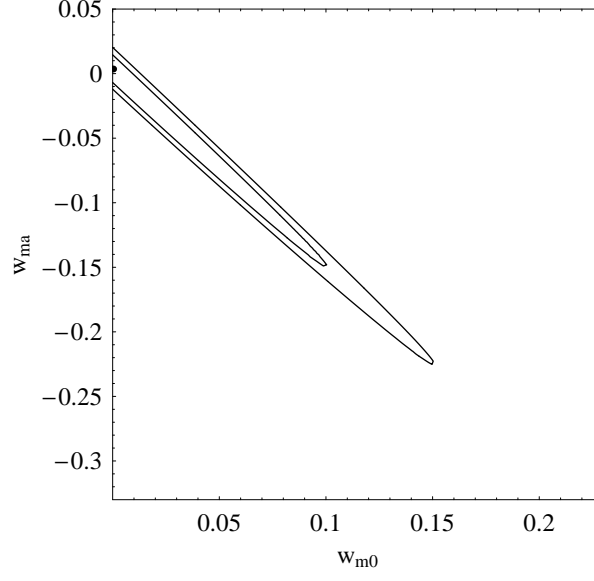


FIG. 1: The 68.3% and 95.4% confidence level contours in the $w_{m0} - w_{ma}$ plane for the AVWDM model. The best-fit parameters are also indicated by a black solid point.

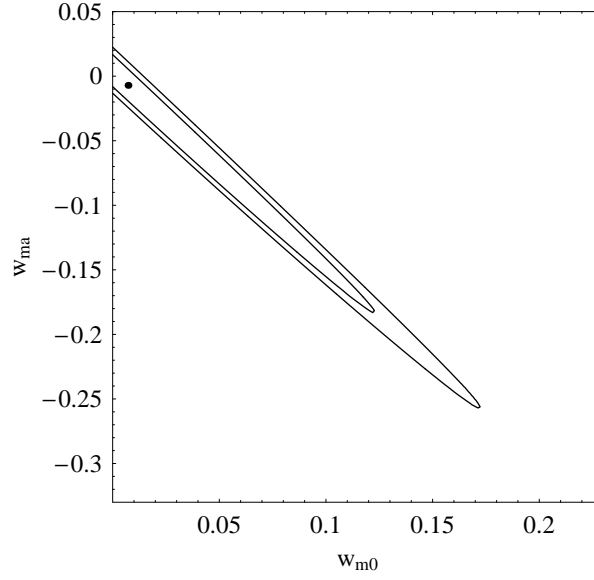


FIG. 2: The 68.3% and 95.4% confidence level contours in the $w_{m0} - w_{ma}$ plane for the XVWDM model. The best-fit parameters are also indicated by a black solid point.

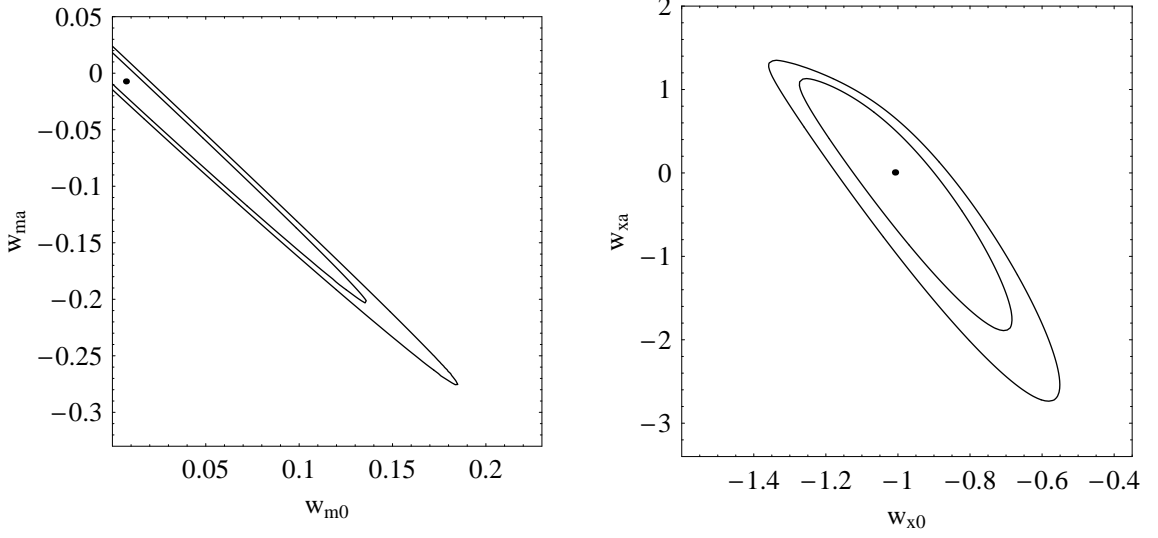


FIG. 3: The 68.3% and 95.4% confidence level contours in the $w_{m0} - w_{ma}$ plane and the $w_{x0} - w_{xa}$ plane for the CVWDM model. The best-fit parameters are also indicated by a black solid point.

At first, we consider the case in which the role of dark energy is played by a cosmological constant Λ . We call it Λ VWDM model. As is well known, the corresponding $E(z)$ is given by [19, 21, 22]

$$E(z) = \left[\Omega_{m0}(1+z)^{3(1+w_{m0}+w_{ma})} \exp\left(-\frac{3w_{ma}z}{1+z}\right) + (1-\Omega_{m0}) \right]^{1/2}. \quad (11)$$

There are 3 free parameters in this model. By minimizing the corresponding total χ^2 in Eq. (8), we find the best-fit parameters $\Omega_{m0} = 0.2774$, $w_{m0} = 0.0$ and $w_{ma} = 0.0036$, while $\chi^2_{min} = 562.227$. In Fig. 1, we present the corresponding 68.3% and 95.4% confidence level contours in the $w_{m0} - w_{ma}$ plane for the Λ VWDM model. It is easy to see that the best fit indicates WDM, while CDM is still consistent with the current observational data. Note that for the best fit, $w_m = 0$ at $z = 0$, namely it is CDM today while it is WDM in the past ($a < 1$).

Next, we consider the case in which the EoS of dark energy w_x is a constant. We call it XVWDM model. In this case, the corresponding $E(z)$ is given by [19, 21, 22]

$$E(z) = \left[\Omega_{m0}(1+z)^{3(1+w_{m0}+w_{ma})} \exp\left(-\frac{3w_{ma}z}{1+z}\right) + (1-\Omega_{m0})(1+z)^{3(1+w_x)} \right]^{1/2}. \quad (12)$$

There are 4 free parameters in this model. By minimizing the corresponding total χ^2 in Eq. (8), we find the best-fit parameters $\Omega_{m0} = 0.2773$, $w_{m0} = 0.0072$, $w_{ma} = -0.0071$, and $w_x = -1.0072$, while $\chi^2_{min} = 562.225$. In Fig. 2, we present the corresponding 68.3% and 95.4% confidence level contours in the $w_{m0} - w_{ma}$ plane for the XVWDM model. Again, it is easy to see that the best fit indicates WDM, while CDM is still consistent with the current observational data. Note that for the best fit, it is always WDM in the whole history ($0 \leq a \leq 1$), and $w_m = w_{m0} = 0.0072$ today. Comparing Figs. 1 and 2, we can easily find that the confidence level contours become larger.

Finally, we consider the case in which the EoS of dark energy w_x is also variable. Here, we adopt CPL parameterization for dark energy, namely

$$w_x = w_{x0} + w_{xa}(1-a), \quad (13)$$

where w_{x0} and w_{xa} are both constants. We call it CVWDM model. In this case, the corresponding $E(z)$ is given by [19, 21, 22]

$$E(z) = \left[\Omega_{m0}(1+z)^{3(1+w_{m0}+w_{ma})} \exp\left(-\frac{3w_{ma}z}{1+z}\right) + (1-\Omega_{m0})(1+z)^{3(1+w_{x0}+w_{xa})} \exp\left(-\frac{3w_{xa}z}{1+z}\right) \right]^{1/2}. \quad (14)$$

There are 5 free parameters in this model. By minimizing the corresponding total χ^2 in Eq. (8), we find the best-fit parameters $\Omega_{m0} = 0.2773$, $w_{m0} = 0.0072$, $w_{ma} = -0.0072$, $w_{x0} = -1.0081$, and $w_{xa} = 0.0062$, while $\chi^2_{min} = 562.225$. In Fig. 3, we present the corresponding 68.3% and 95.4% confidence level contours in the $w_{m0} - w_{ma}$ plane and the $w_{x0} - w_{xa}$ plane for the CVWDM model. From Fig. 3, we see that Λ CDM is still consistent with the current observational data. Note that for the best fit, it is always WDM in the whole history ($0 < a \leq 1$), and $w_m = w_{m0} = 0.0072$ today.

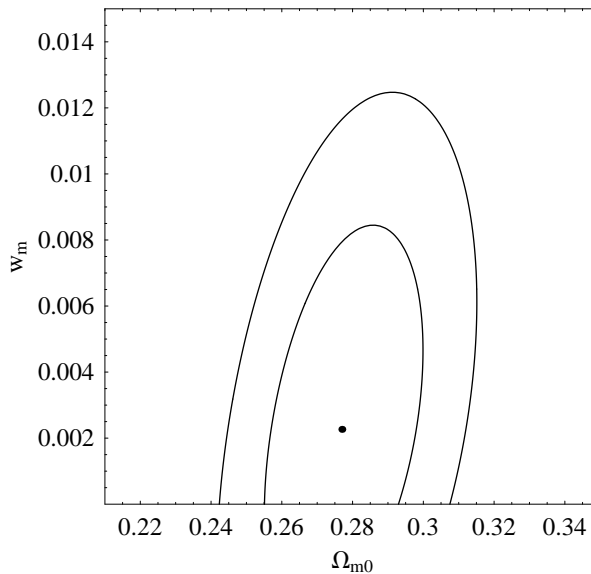


FIG. 4: The 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - w_m$ plane for the Λ WDM model. The best-fit parameters are also indicated by a black solid point.

IV. COSMOLOGICAL CONSTRAINTS ON WDM WITH CONSTANT EOS

Noting that in most of the previous works constant EoS of both WDM and dark energy are assumed, here we also consider WDM whose EoS w_m is constant, but dark energy is a cosmological constant Λ , or dark energy whose EoS are constant and CPL parameterized, respectively. We try to see whether the cosmological constraint on the constant EoS of WDM is robust for various types of dark energy, especially when the EoS of dark energy is variable.

We firstly consider the case with a cosmological constant Λ . We call it Λ WDM model. Note that the EoS of WDM should satisfy $0 \leq w_m \leq 1/3$ as mentioned in Sec. I. As is well known, the corresponding $E(z)$ is given by [19, 21, 22]

$$E(z) = \left[\Omega_{m0}(1+z)^{3(1+w_m)} + (1-\Omega_{m0}) \right]^{1/2}. \quad (15)$$

There are 2 free parameters in this model. By minimizing the corresponding total χ^2 in Eq. (8), we find the best-fit parameters $\Omega_{m0} = 0.2770$ and $w_m = 0.0023$, while $\chi_{min}^2 = 562.228$. In Fig. 4, we present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - w_m$ plane for the Λ WDM model. It is easy to see that the best fit indicates WDM, while CDM is still consistent with the current observational data.

Then, we turn to the case in which the EoS of dark energy w_x is a constant. We call it XWDM model. In this case, the corresponding $E(z)$ is given by [19, 21, 22]

$$E(z) = \left[\Omega_{m0}(1+z)^{3(1+w_m)} + (1 - \Omega_{m0})(1+z)^{3(1+w_x)} \right]^{1/2}. \quad (16)$$

There are 3 free parameters in this model. By minimizing the corresponding total χ^2 in Eq. (8), we find the best-fit parameters $\Omega_{m0} = 0.2776$, $w_m = 0.0025$, and $w_x = -1.0035$, while $\chi_{min}^2 = 562.225$. In Fig. 5, we present the corresponding 68.3% and 95.4% confidence level contours in the $w_m - w_x$ plane for the XWDM model. Again, the best fit indicates WDM, while CDM is still consistent with the current observational data.

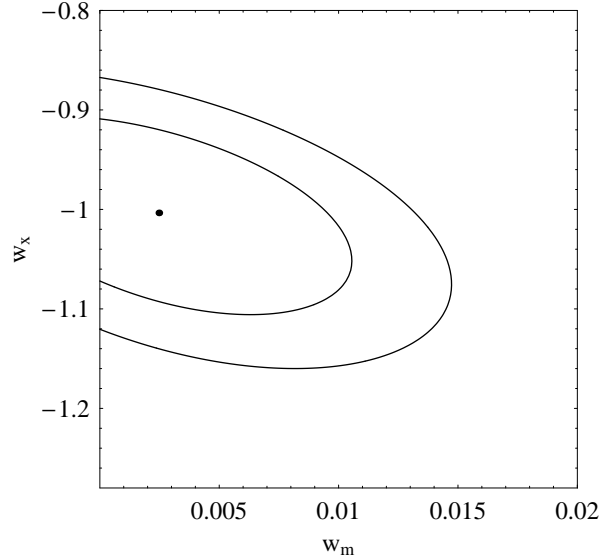


FIG. 5: The 68.3% and 95.4% confidence level contours in the $w_m - w_x$ plane for the XWDM model. The best-fit parameters are also indicated by a black solid point.

Finally, we consider the case with CPL parameterized dark energy, namely, the EoS of dark energy is given by Eq. (13). We call it CWDM model. In this case, the corresponding $E(z)$ is given by [19, 21, 22]

$$E(z) = \left[\Omega_{m0}(1+z)^{3(1+w_m)} + (1 - \Omega_{m0})(1+z)^{3(1+w_{x0}+w_{xa})} \exp\left(-\frac{3w_{xa}z}{1+z}\right) \right]^{1/2}. \quad (17)$$

There are 4 free parameters in this model. By minimizing the corresponding total χ^2 in Eq. (8), we find the best-fit parameters $\Omega_{m0} = 0.2773$, $w_m = 0.0023$, $w_{x0} = -1.0074$, and $w_{xa} = 0.0299$, while $\chi_{min}^2 = 562.224$. In Fig. 6, we present the corresponding 68.3% and 95.4% confidence level contours in the $w_m - w_{x0}$ plane and the $w_m - w_{xa}$ plane for the CWDM model. From Fig. 6, we see that Λ CDM is still consistent with the current observational data.

Comparing these three cases, it is easy to find that their best-fit w_m are almost the same value. This indicates that the cosmological constraint on the constant EoS of WDM is fairly robust, namely it is insensitive to the dark energy models.

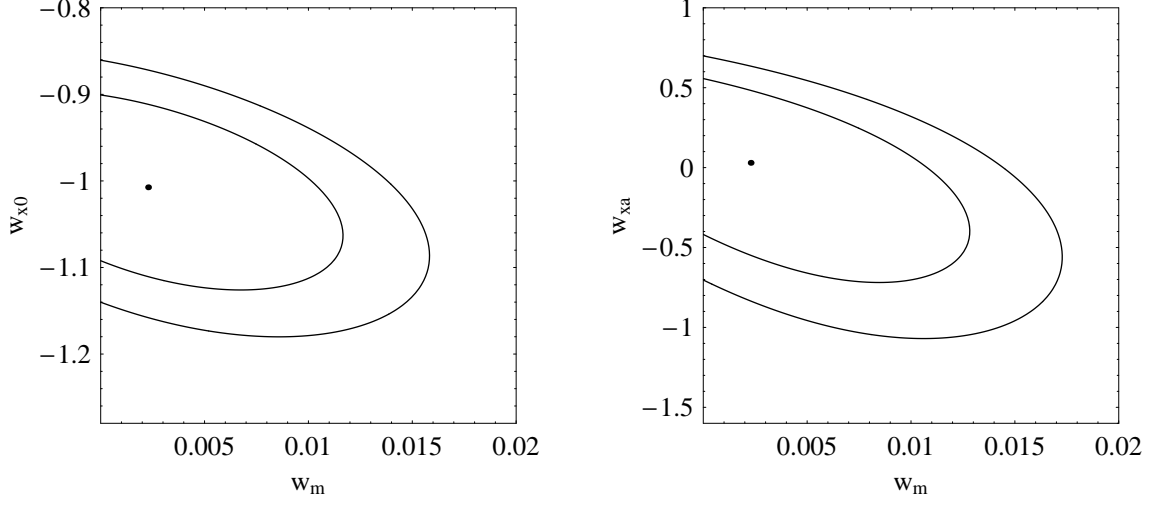


FIG. 6: The 68.3% and 95.4% confidence level contours in the $w_m - w_{x0}$ plane and the $w_m - w_{xa}$ plane for the CWDM model. The best-fit parameters are also indicated by a black solid point.

V. CONCLUDING REMARKS

Although Λ CDM model is very successful in many aspects, it has been seriously challenged. Recently, warm dark matter (WDM) remarkably rose as an alternative of cold dark matter (CDM). In the literature, many attempts have been made to determine the equation-of-state parameter (EoS) of WDM. However, in most of the previous works, it is usually assumed that the EoS of dark matter (DM) is constant (and usually the EoS of dark energy is also constant). Obviously, this assumption is fairly restrictive. It is more natural to assume a variable EoS for WDM (and dark energy). In the present work, we try to constrain the EoS of variable WDM with the current cosmological observations. We find that the best fits indicate WDM, while CDM is still consistent with the current observational data. On the other hand, in this work we also consider WDM whose EoS is constant, while the role of dark energy is played by various models. We find that the cosmological constraint on the constant EoS of WDM is fairly robust.

As mentioned above, in the six cases considered in this work, all the best fits indicate WDM, while CDM is still consistent with the current observational data. So, it is worthwhile to compare these WDM models with Λ CDM. To this end, we also fit Λ CDM model to the same observational data, and find the best-fit $\Omega_{m0} = 0.2736$, while $\chi^2_{min} = 562.546$. Following [19, 20], here we adopt three criterions used extensively in the literature, namely χ^2_{min}/dof , Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC). Note that the degree of freedom $dof = N - k$, whereas N and k are the number of data points and the number of free model parameters, respectively. The BIC is defined by [23]

$$\text{BIC} = -2 \ln \mathcal{L}_{max} + k \ln N, \quad (18)$$

where \mathcal{L}_{max} is the maximum likelihood. In the Gaussian cases, $\chi^2_{min} = -2 \ln \mathcal{L}_{max}$. Thus, the difference in BIC between any two models is given by

$$\Delta \text{BIC} = \Delta \chi^2_{min} + \Delta k \ln N. \quad (19)$$

On the other hand, the AIC is defined by [24]

$$\text{AIC} = -2 \ln \mathcal{L}_{max} + 2k. \quad (20)$$

Correspondingly, the difference in AIC between any two models reads

$$\Delta \text{AIC} = \Delta \chi^2_{min} + 2\Delta k. \quad (21)$$

Model	Λ CDM	Λ VWDM	XVWDM	CVWDM	Λ WDM	XWDM	CWDM
χ^2_{min}	562.546	562.227	562.225	562.225	562.228	562.225	562.224
k	1	3	4	5	2	3	4
χ^2_{min}/dof	0.968238	0.971031	0.972708	0.974393	0.969359	0.971028	0.972706
Δ BIC	0	12.4139	18.7784	25.1449	6.04847	12.4119	18.7774
Δ AIC	0	3.681	5.679	7.679	1.682	3.679	5.678
Rank	1	4	6	7	2	3	5

TABLE I: Comparing all the six WDM models with Λ CDM.

In Table I, we present χ^2_{min}/dof , Δ BIC and Δ AIC for Λ CDM and all the six WDM models considered in this work. Notice that Λ CDM has been chosen to be the fiducial model when we calculate Δ BIC and Δ AIC. From Table I, we see that the rank of models is coincident in all the three criterions (χ^2_{min}/dof , BIC and AIC). Obviously, Λ CDM model is the best one. In summary, although the best fits to the cosmological observations (SNIa, CMB and BAO) indicate WDM, we cannot say WDM is favored, since Λ CDM is still better than WDM models from the viewpoint of χ^2_{min}/dof , BIC and AIC. So, in order to distinguish WDM and CDM, the further observations on the small/galactic scale are required.

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